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ABSTRACT

This paper continues the analysis of data from 3-year case studies of two United Kingdom schools presented in an earlier edition of the Journal of Research in Mathematics Education (v29, n1) in order to illustrate the use of a particular situated method of interpretation. Various classroom incidents are analyzed, demonstrating that the constraints and affordances of formalized mathematics classrooms to which students become attuned contribute to the development of learning identities that are peculiar to the school mathematics classroom but are of limited use to students in the "real world." An understanding of the mathematics classroom as a particular community of practice is central to this analysis. Contains 40 references. (Author/NB)

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The Community of the Mathematics Classroom: Situated Insights into Knowledge Development and Use

by
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The Community of the Mathematics Classroom: Situated insights into knowledge development and use.

Jo Boaler, Stanford University

Abstract

In this paper I continue my analysis of data from 3-year case studies of two schools, presented in an earlier edition of JRME, (29, 1), in order to illustrate the use of a particular situated method of interpretation. Various classroom incidents are analysed, demonstrating that the 'constraints and affordances' (Gibson, 1986; Greeno & the Middle School Mathematics Through Applications Project Group (MMAP), 1998) of formalized mathematics classrooms, to which students become attuned, contribute to the development of learning identities that are peculiar to the school mathematics classroom and of limited use to students in the "real world". An understanding of the mathematics classroom as a particular community of practice is central to this analysis.

Introduction

The influence of the environment upon knowledge development and use has, until recently, received relatively little attention in the field of education, as individual, cognitive representations of knowledge have pervaded theories and practices of learning. But we are now, as Resnick has claimed 'in the midst of multiple efforts to merge the social and cognitive' (1993, p3), which has meant that significant attention is being paid to the nature of situations and contexts and the 'negotiation of people with each other and with the resources of their environment' (Greeno & MMAP, 1998, p9). Theories of situated cognition have added a fresh and radical perspective to debates about knowledge, particularly in their questioning of the existence of knowledge transfer, an idea upon which most educational institutions are predicated. This has been replaced, within situated theory, with the idea that knowledge is emergent within, and co-constituted by, the different environments in which it is produced. But the situated perspective is still in its infancy and whilst important contributions have been made to further our understandings of this perspective (Lave, 1988; Brown, Collins & Duguid, 1989; Hennessy, 1993; Young, 1993; Greeno & MMAP, 1998), knowledge of the precise ways in which situated perspectives can inform events within mathematics classrooms remains elusive.

My aim in this paper is to show how the perspective of situated cognition may illuminate the complexities of students' mathematical behavior in school and real world settings. In a previous edition of JRME (29, 1) I described the results of a 3-year study of two schools that taught mathematics in totally different ways. One school used transmission methods of teaching that encouraged students to develop learning identities that were extremely ineffective in non-classroom settings. The other school used project-based methods of teaching that appeared to encourage learning identities that were consistent with the demands of both the classroom and the real world. My analysis in that paper, which followed on from descriptions of the two schools and the students' performance on various assessments, focussed only upon the different forms of knowledge that students appeared to have developed. I would like in this paper to start from the results of that study in order to extend and, in

places modify my original interpretations. My aim in doing so is to show the ways that a situated perspective, more than others that have gone before, is sufficiently broad and nuanced to inform the relationships formed between learners, their mathematical understanding and the environments in which they work.

Theories of Situated Cognition.

Situated perspectives differ from many others that have gone before them, in their focus upon broad activity systems (Greeno & MMAP, 1998) or communities of practice (Lave, 1988). Most distinctively, situativity locates learning as a social and cultural activity and success is not focused upon the cognitive attributes that individuals possess, but upon the ways in which those attributes play out in interaction with the world. This perspective has significant implications for educational theory and practice. Previous theories of learning that have been applied to students of mathematics, such as behaviorism and cognitivism, have focused upon individuals and upon mathematics, with the former concentrating upon the repetition of appropriate mathematical behaviors, and the latter concentrating upon the 'acquisition' (Sfard, 1998, p5) of mathematical knowledge and understanding. Situated perspectives suggest that the behaviors and practices of students in mathematical situations are not solely mathematical, nor individual, but are emergent as part of the relationships formed between learners and the people and systems of their environments. Notions of learning transfer have been rejected within situated theories, not as Anderson, Reder, & Simon (1996) have suggested, because individuals do not make use of knowledge gained in one setting in another, but because the knowledge that is used in a new setting is always created in and for that setting. Individuals may draw upon previous memories of events when they are in new settings, but the knowledge that is constructed in that setting always emerges as a product of the different relations between the people and systems of their particular environment. As Hutchins has proposed, 'the properties of the interaction between individual minds and artifacts of the world', (italics added) are at the essence of human performance (1993, p62).

Greeno & MMAP (1988) have proposed the use of a particular method of situated analysis that considers the constraints and affordances provided by different learning environments (Gibson, 1986) and students' attunement to these. As an example, Greeno & MMAP apply the analysis of constraints and affordances to Lave's (1988) shoppers, who were asked to work out which of two products was a "better buy". The shoppers' response to this problem could be taken as an indication of mathematical ability, but in this situated analysis the adults' mathematical ability or knowledge, is only one aspect of their participation or performance. Instead, the focus of the analysis is on the interaction of the adults' abilities with constraints and affordances of their environment. Affordances in this situation include the layout of the store and the numerical information given about prices, whereas constraints include the amount of food the adults needed, the amount of space they had at home and the costs of different products. These different constraints and affordances all impacted upon the shoppers' responses to the problems. In Greeno & MMAP's model the regularities of a person's practice over time are not thought of in terms of narrow conceptions of ability but by their attunement to the different constraints and affordances provided by their environments.

Situated perspectives turns the focus away from individual attributes and towards broader communities. The implication of this shift for research within mathematics education is a focus upon the communities or activity systems generated within mathematics classrooms, which are not distinguished from the cognitive processes that students produce within such systems. Lave & Wenger (1991) have provided an analysis of apprenticeship learning and relate the process of learning as an apprentice as one of moving from legitimate to full participation within established communities. There are a number of important differences between learning as an apprentice and as a school student of mathematics, to which I shall return, but Lave & Wenger's description of a community as 'participation in an activity system about which participants share understandings concerning what they are doing and what that means for their lives' (1991, p98) pertains in many ways to the school mathematics classroom, as I hope to indicate. In my previous analysis of the two schools I took a largely individual perspective upon the students' development of knowledge, but I have since found it useful to take the broader community as the unit of analysis, because a focus upon 'intact activity systems' (Greeno & MMAP, 1998, p5), rather than individuals within those systems, has given more complex and subtle insights into the mathematical behavior of both groups and individuals in the two schools.

Research Design

In order to investigate the knowledge that students develop in school and the extent to which they are able to make use of school knowledge, I conducted three-year case studies of a cohort of students in two UK schools. These longitudinal case studies combined a variety of qualitative and quantitative methods, but for the sake of brevity, I shall refer readers back to my original paper for a full description of research methods and design (JRME, 29, 1). Similarly, I will not attempt to repeat the descriptions of the two schools, the teaching methods employed and the cohorts of students who attended the schools, given in my earlier paper. Instead I offer the following summary of the two schools and the main results of the study.

The Two Schools

At 'Amber Hill' school there was approximately 200 students in the year group I followed and the students were taught mathematics using a traditional, textbook approach (Romberg & Carpenter, 1986). The teachers explained methods and procedures from the chalkboard at the start of lessons, and the students then practiced the procedures in textbook questions. In this school the students were grouped into sets 1-8, according to the teachers' perceptions of ability. At 'Phoenix Park' school there was approximately 110 students in the year group, the students were taught in mixed-ability groups and teachers introduced the students to different ideas and problems that could be investigated and solved using applied mathematical methods. Students worked in pairs or groups of their choosing and they discussed and negotiated mathematical directions and solutions with each other and with their teachers. There were no books or schemes used in the school, until a few weeks before the students took the national school leaving examination — the General Certificate of Secondary Education (GCSE) — when they were given examination questions to practice and they were introduced to formalized mathematical methods and notations. Prior to the beginning of my study, students at both schools had spent two years following the

same mathematics approach. Both schools were situated in mainly white, working-class areas and there were no significant differences in the cohorts in terms of sex, ethnicity, social class or prior mathematical attainment (see Boaler, 1997).

The differences in the students' experiences over the three years had a huge impact upon their mathematical perceptions and behaviors. At Amber Hill school the students worked hard and completed a great many textbook exercises, but many of the students developed the perception that school mathematics was made up of numerous rules, formulas and equations, that needed to be memorized. They believed that their role in the mathematics classroom was to memorize the different rules they had been given and many believed that school mathematics was incompatible with thought. The students' beliefs were consistent with those that Schoenfeld (1988) offers as being typical responses of students taught using traditional instruction methods. In contrast, the Phoenix Park students took a very relaxed approach to work, but when they did work it was because they wanted to. As part of their project work they learned to choose, adapt and apply mathematical methods and they developed the idea that mathematics was a thinking, flexible subject. The degrees of satisfaction the two sets of students developed in relation to school mathematics were similar, but their views about the nature of mathematics were not.

In two different project-based assessments I gave the students when they were in years 10 and 11, the Phoenix Park students outperformed the Amber Hill students; in more closed and procedural mathematics questions, given to the students in years 9, 10 and 11 there were never any differences between the two groups of students. In the GCSE examination, which is made up of procedural and conceptual mathematics questions in an approximate ratio of 2:1, the Phoenix Park students attained significantly higher grades. Eighty-eight per cent of the Phoenix Park cohort passed the examination compared with 71% of the Amber Hill cohort, ($\chi^2 = 22.22$, d.f. = 1, $n = 290$, $p < 0.001$). At Amber Hill, students solved approximately half as many conceptual questions as procedural questions; at Phoenix Park students solved equal proportions of each type (see Boaler, 1998b).

Thus, although the Amber Hill students spent more time on task (Peterson & Swing, 1982; Boaler, 1997) in lessons and completed a lot of textbook work, whilst the Phoenix Park students spent a large proportion of their lessons wandering about the room or chatting, it was this latter group of students who were more able to use mathematics in a range of settings. Much of the evidence for the differences between the students in the two schools is provided elsewhere (Boaler, 1997, 1998a) because I would like in this paper to concentrate upon the community behavior and identities that were developed at the two schools that had an important impact upon their knowledge development and use. To illustrate what I mean by this I shall use a combination of lesson observations, interviews and assessments.

The Artificial School.

Scribner and Cole (1981) propose that the majority of what students learn in school is the appropriate way to behave in school. Central within this idea, and situated theory more generally, is the notion of community. It was clear at Amber Hill school that the students had developed a firm sense of the school mathematics classroom and the behaviors and practices that were appropriate within that community. These derived from the commonalties of their classroom, textbook experiences and they became apparent when students encountered situations that

deviated in some way from the normal classroom patterns. One set of classroom practices, that I described in my previous paper, was a form of 'cue based' (Schoenfeld, 1985) behavior that the students used to determine choice of mathematical method. This behavior was an important indication of the shared, community expectations and practices that students developed and that were specific to the mathematics classroom. For example, the following notes were taken from my observations of a year 10 class:

> Lina calls me over and says that she is 'stuck'. She needs to start a new exercise in the textbook and says that she does not know what to do. I explain the first question to her and she says 'but this is the same'. I look at the previous exercise and find that there is indeed no difference between the demands of that exercise and the new one Lina is about to move onto.

I have described this sort of behavior as 'cue based' because Lina stopped herself from moving onto the next exercise because the demand of the new exercise was different to what she expected. Lina's confusion was not 'mathematical', she possessed the appropriate knowledge of mathematics to answer the question, but she was used to interpreting non-mathematical 'cues' as an indication of what to do. One of these cues was when moving onto a new exercise, use a more difficult procedure. This sort of behavior was common amongst the Amber Hill students. The students had learned to expect later exercises in a book to become more difficult, so they would often use a more complicated procedure even if this was inappropriate. Other examples of 'cue based' responses included the following:

- When working through exercises, students expected to use the method they had just been taught on the board. If a question required the use of a different method, they would often get the answer wrong, or become confused and ask for help. This occurred even when students knew how to use the required method.
- If a question required some real world knowledge, or non-mathematical knowledge — for example a question I observed in which the students had to say why there are more females than males in the population — students would stop and ask for help. They would be able to answer the question if prompted, they would probably be able to answer the question if they were in a science classroom, or if they were at home, but their expectation of the knowledge they should use in a mathematics classroom stopped them from answering such questions.
- The students always expected to use all of the numbers given to them in a question, or all of the lines present on a diagram. If students didn't use them all they thought they were doing something wrong and changed their methods to ones that could include all of the numbers or lines.

These different examples demonstrate the codes of the mathematics classroom that students lived by, the implicit practices of that community that influenced their use of mathematics. If students encountered textbook situations that departed from their expectations, they became confused, not because of the extent of their mathematical 'knowledge', but because of the regularities of the mathematics classroom to which they had become attuned. Consideration of the nature of these classroom regularities, which may be interpreted as particular constraints and affordances for the students, makes this behavior easy to understand.

In mathematics lessons the students would be given demonstrations of mathematical procedures which they practiced in textbook exercises. They rarely ever had to make choices about mathematical procedures, because they were always expected to follow the procedure they had just been taught on the board. In addition, the

textbook questions always presented the exact information students needed to answer a question; students never had to select the information they needed. Also, all of the exercises would be graded in order of difficulty, so, for example, if students had to multiply two numbers in the first exercise, they knew they would have to, for example, multiply three numbers in the next or divide two numbers. The students came to regard these regularities as affordances that helped them work through their textbook questions with ease. The students became proficient at interpreting these different textbook cues, rather than employing mathematical thought or sense making.

Another important affordance that students came to rely upon within the classroom was the help of the teacher. If students at Amber Hill were unsure how to answer a question and they could not find or interpret a cue to help them, they would ask the teacher. In such instances the teacher would invariably tell the students which method to use, demonstrating the procedure that students needed to follow. The students would then use this procedure throughout the rest of the exercise. In this way the teachers and students conspired to make mathematical thought redundant. Another affordance that students used was the worked example in the textbook that they would repeat with different sets of numbers. The students also adapted to a set of classroom constraints, such as the need to complete each exercise in a certain amount of time in order to keep up with the rest of the class. This constraint meant that they would never spend more than a few moments on any one question in an exercise. All of these different constraints and affordances were important to the students' mathematical behavior, yet they were clearly related in intrinsic ways, to the students' classroom environment, rather than their cognitive attributes. The answers students gave in textbook and other situations could be considered to be a representation of their mathematical knowledge, but the students' responses at Amber Hill seemed to represent more accurately the students' attunement to the particular constraints and affordances of their school environment. Even when students 'possessed' appropriate knowledge, they were inhibited from making use of it because of the particular situation within which they were working. In my previous interpretation I suggested that the forms of knowledge the students developed at Amber Hill were inadequate, but this analysis is insufficient, because it is based upon the students' individual cognitive attributes, paying insufficient attention to the ways that their cognitive attributes were mediated by their classroom environment. The students' behavior as they worked through their textbook exercises gave a clear indication of the situated nature of their learning and the impact of the norms of their mathematics classroom upon their emergent mathematical knowledge.

Another example of the situated nature of their learning was provided by the students' responses to an applied 'architectural activity' that I gave them. As part of this activity students had to estimate a simple angle. Three-quarters of the students in the highest ability group at Amber Hill tried to use trigonometry to calculate the angle exactly. The students had just been taught about trigonometry and the word 'angle' appeared to cue them into the use of these methods, even though they were inappropriate in the context of the activity (and the students used them incorrectly). If they had encountered this problem outside school it is unlikely that the students would have attempted to use trigonometry in this way.

A third example of the way in which classroom environments impacted upon the students' use of mathematics was provided by another applied activity I gave to the students. In this activity students were given a scale plan of an apartment that showed external walls and windows and they were asked to design an apartment

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for themselves, inserting walls, doors and furniture. They were then asked to find the approximate cost of carpeting the whole apartment, with a particular carpet, the price of which was given, per linear metre. Thirty-four per cent of students at Amber Hill (n = 99) ignored the request to find an approximate cost of carpet and worked out the exact area of floor space, subtracting the area of any protrusions into the rooms, such as fireplaces or bay windows. But the area of room protrusions cannot be subtracted from the overall length of carpet bought, which meant that the students calculated insufficient amounts of carpet for their apartments. I interviewed students after they had completed this activity and asked them why they chose to work with such a degree of accuracy. The students reported that they would not have done so in a real life situation, but because they were in their mathematics classroom, they felt the need to 'show their math' and work with as much accuracy as possible. This line of thinking, which could also account for the students' response to the architectural activity, was interesting because the students showed that their choice of mathematical procedure was not determined by mathematical sense making but by the context of the mathematics classroom and the constraints and affordances to which they had become attuned.

The students' responses to textbook exercises and applied assessments demonstrated their attunement to particular classroom constraints and affordances that were peculiar to the school mathematics classroom. This emerged as a key limitation for the students during the course of my study. The GCSE examination, which the students took when they were 16, provided a different set of constraints and affordances and this limited the students' performance. For example, the examination questions did not provide cues as to which method to use and the teacher help that students often depended upon was absent. When I asked the Amber Hill students about their experience of the examination the students reported that their attunement to these particular classroom affordances had detracted from their examination performance (Boaler, 1997, 1998a).

The students' attunement to the constraints and affordances of their mathematics classroom also had enormous implications for the students' use of mathematics in the real world. The students reported that in their jobs and everyday lives they faced completely different sets of constraints and affordances. The realisation of this difference between the environments of school and the real world caused the students to believe school mathematics to be an irrelevance and they developed the idea that their school mathematics knowledge had boundaries (Siskin, 1994) or barriers surrounding it, that kept it firmly within the mathematics classroom. When I interviewed 40 of the students I asked them if they used mathematics outside school. They all said that they did — many of them had part-time jobs at that stage — but when I asked the students if they made use of methods they learned in school, they all said that they did not. The students talked about their perceptions of school mathematics as a strange and specialised type of code that they would only use in one place — the mathematics classroom. What was particularly interesting was the fact that the students all offered different reasons why they would not make use of school mathematics, but these all related to the differences between the environments of school and the real world. The students did not describe mathematical differences, or the problems of their mathematics teaching — which they were happy to talk about at other times — they described the importance of situations outside school, the lack of complication, the social nature of the 'real world' and being separated from the partners with whom they worked in lessons. For example:

G: I use my own methods.

JB: Why is that do you think?

G:'Cause when we're out of school yeah, we think, when we're out of school it's social, you're not like in school, it tends to be social, so it would be like too much change to refer back to here.

(George, Amber Hill, Y10)

The differences the students described, which caused them to abandon their use of school learned methods, related to the constraints and affordances of the real world, and the presence of different people and systems in the environment. It was clear from the students' descriptions that their use of mathematics in situations within and outside school was driven by the different environments, and whilst their individual cognitive attributes may have been important, these were not the only factors influencing their use of mathematics.

In my previous analysis of the inadequacy of students' knowledge at Amber Hill school I overlooked an important point that is central within situated perspectives. That is, the Amber Hill students were effective participants of the community of their school mathematics classroom. The students only experienced problems because this effectiveness did not 'transfer' elsewhere, as one of the student's described:.

A: It's stupid really 'cause when you're in the lesson, when you're doing work — even when it's hard — you get the odd one or two wrong, but most of them you get right, and you think well, when I go into the exam I'm gonna get most of them right, 'cause you get all your chapters right. But you don't. (Alan, AH, year 11)

Being effective in the classroom community involved a strict adherence to school and mathematical rules, the interpretation of non-mathematical cues and the suppression of thought. All of these practices that became part of the students' learning identities, are incompatible with authentic activity. These practices were not deliberately encouraged by the teachers of mathematics at Amber Hill, but they gradually developed amongst the community of learners, and practices that were rare at the beginning of year 9 were well established by the end of year 11. Awareness of these common behaviours, that were shared within the school mathematics community, increased my understanding of the students' mathematical practices within textbook, applied, GCSE and real world situations.

School for Life.

The constraints and affordances the students experienced at Phoenix Park were very different from those of Amber Hill School. When the Phoenix Park students approached a problem, they faced the constraint of not being able to ask the teacher which method they should follow (as teachers would not give this sort of structured help), but the students used the discussions they held with each other as affordances that helped them to develop mathematical directions. Other affordances that students were encouraged to make use of were mathematics books, dictionaries, computers and calculators that they were allowed to use as a resource and the school library to which they had access during lessons. However, the students were not able to rely upon textbook regularities or structured demonstrations of method and they were forced to develop learning identities that included a propensity to think for themselves and a willingness to adapt and change mathematical methods. The affordances the students used at Phoenix Park did not only allow them to think and make flexible use of mathematics

methods, they required them to think and, with that, develop mathematical understanding. The affordances the Amber Hill students came to rely upon precluded thought encouraging only an esoteric set of practices that were specific to the mathematics classroom. As the Phoenix Park students worked on their projects they were only introduced to methods that became relevant to their particular chosen themes, but the depth of understanding they developed of the methods they used, combined with their experience of adapting methods to fit new situations, appeared to compensate for the restricted range of methods they had met.

When I gave students at both schools the same assessments, the impact of the different constraints and affordances of the two school environments became clear. In the architectural activity that I have already described, 75% of Phoenix Park students were successful, compared with 55% of Amber Hill students. In the apartment design activity that I have described, 61% of Phoenix Park students were successful, compared with 31% of Amber Hill students (see Boaler, 1997). Part of the reason for the Phoenix Park students' relative success on both these activities was the similarity of the constraints and affordances provided by the applied activities and their classroom projects. In their projects the students had learned to consider the situations they had been given, think about the information they needed to find and choose the most appropriate mathematical methods to help them, they had not been trained to look for cues that may indicate the appropriate method to use. When the students were asked to work out the approximate cost of carpet, the Amber Hill students gave a clear indication of the situated nature of their learning, by choosing to work with the maximum degree of accuracy, because they were in a mathematics classroom. But the fact that the Phoenix Park students chose to interpret what was required in the question and act accordingly was also an indication of the situated nature of their learning. Both sets of students were responding to the constraints and affordances of their school environments to which they had become attuned.

The independence and freedom the Phoenix Park students experienced in school also served as a particular affordance in some mathematical situations which further distinguished their behaviour from that of the Amber Hill students. For example, in the apartment design activity, the students had been asked to choose and plan the rooms they would have in their apartment. All of the students in both schools included in their designs at least one bedroom, bathroom, living room and kitchen. At Amber Hill only 6 of the 99 designs (6%) included a more unusual room, with 2 poolrooms, 2 rooms with swimming pools, 1 playroom and 1 storeroom. Amongst the 89 Phoenix Park designs there were 35 examples of 'unusual' rooms (39%) with: 7 games rooms, 4 soccer rooms (generally including small 5-a-side pitches), 3 indoor swimming pools, 3 studies, 2 hi-fi rooms, 2 children's playrooms, 2 cocktail bars and one each of a 'bouncy castle' room, a pool room, a Jacuzzi, a computer room, a gym, a garage, a bowling alley, a utility room, a piano room and a disco room. The Phoenix Park students appeared to demonstrate the more natural responses of 14-year olds when asked to design their own living space. The Amber Hill students included the rooms that they thought that they should have, the rooms that they thought they should design in response to a school mathematics task. The Amber Hill students' responses were constrained by their perceptions of what constituted 'academic work' (Doyle, 1988, p168) and co-constituted by the environment in which they were produced.

There were many other indications of the students' different interpretations of what was appropriate in a mathematics classroom. At one time I observed a Phoenix Park student finish his project description, paste it onto colored paper and carefully cut the paper out into the shape of a dinosaur. He did this because he wanted to, but this was not an event that would have taken place within an Amber Hill mathematics classroom. Perhaps that is a good thing, after all there is little mathematics involved in the cutting out of a dinosaur shape, but the freedom the Phoenix Park students experienced to work in this way contributed to their perception that the mathematics classroom was like other environments within their experience. Such perceptions stopped the students from thinking that school mathematics was a specialised and artificial language. The students at Phoenix Park became attuned to classroom constraints and affordances in the same way as the Amber Hill students did, but the Phoenix Park students' constraints and affordances were more consistent with those of their lives.

In the GCSE examination, the students at Amber Hill found that the classroom constraints and affordances to which they had become attuned were not represented, but the Phoenix Park students' attunements proved to be much more helpful. The GCSE examination presents a series of short questions, many of which require the use of formalised methods. The Phoenix Park students experienced the constraint of having met few of these methods, but this constraint was balanced by their understanding of the methods they had met, their willingness and ability to adapt and change methods to fit new situations and their propensity to think for themselves:

T: I think it allows — when you first come to the school and you do your projects, it allows you to think more for yourself than when you were in middle school and worked from the board or from books. (Which) helped with the exams where we had to — had to think for ourselves there and work things out. (Tina, PP, year 11)

In my original paper I used this quote to illustrate the 'ability' of the Phoenix Park students to think for themselves. But both sets of students should have been able to think for themselves in the examination. The reason the Amber Hill students were inhibited from doing so was because thinking for themselves had not been a part of their classroom practice. The Phoenix Park students were willing to think in the examination because of their experiences in the classroom. Thinking for themselves was a practice to which the Phoenix Park students became attuned and which became a part of their broad learning identities. This helped 88% of the cohort to pass the mathematics examination — which is higher than the national average, even though the attainment of the cohort was lower than the national average when students started their project-based approach.

In the real world the Phoenix Park students experienced further advantages. When I interviewed the students and asked them questions about their use of school methods or their own methods, over three-quarters of the students reported that they made use of the mathematics they learned in school. This was because they did not see a real difference between the mathematics of school and the 'real world', as the students described:

L: Yeah when we did percentages and that, we sort of worked them out as though we were out of school using them.

V: And most of the activities we did you could use.

L: Yeah most of the activities you'd use — not the actual same things as the activities, but things you could use them in.

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(Lindsey and Vicky, PP, year 11)

JB: And when you use math in situations outside of school do you use the methods you have learned in school or do you tend to use your own?

T: Use those maths what I've learned here. (Tina, PP, year 11)

G: When I'm out of school now, I can connect back to what I done in class so I know what I'm doing.

JB: What do you think?

J: It just comes naturally, once you've learned it you don't forget.

(Gavin and John, PP, year 10)

D: Probably try and think back to here and maybe try and think of my own methods sometimes, depending what sort of situation.

JB: So you would think back here for some things?

A: Yes it would be really easy to think back here.

JB: Why do you think that?

A: I don't know, I just remember a lot of stuff from here, it's not because it wasn't long ago, it's just because, it's just in my mind.

(Danny & Alex, PP, Y 11)

Although the students at the two schools only gave their reports of their use of mathematics, these reports were consistent with the mathematical behaviour they demonstrated in other situations. The Amber Hill students' descriptions indicated that they saw little use for the mathematics they learned in school in out-of-school situations and so they abandoned their school-learned mathematics and invented their own methods. The students appeared to regard the worlds of the school mathematics classroom and the rest of their lives as inherently different. This was not true for the Phoenix Park students who did not think of their school mathematics knowledge as an irrelevant set of practices bounded within the mathematics classroom.

Further evidence for the broad influence of the mathematical environments students worked within and the identities they developed in relation to these, was provided by the Phoenix Park students' reflections upon a different kind of mathematics environment that they experienced in the weeks prior to their GCSE examination. At that time the students had been taken off their project work and introduced to formal examination procedures that they practiced in textbook and examination questions. Sue was in the midst of this examination preparation when I asked her the following question:

JB: Do you think when you use math outside of school, it feels very different to using math in school, or does it feel similar?

S: Very different from what we do now, if we do use math outside of school it's got the same atmosphere as how it used to be, but not now.

JB: What do you mean by — "it's got the same atmosphere"?

S: Well, when we used to do projects, it was like that, looking at things and working them out, solving them — so it was similar to that, but it's not similar to this stuff now, it's, you don't know what this stuff is for really, except the exam. (Sue, PP, year 11)

The Phoenix Park students believed that the mathematics they learned through their project work was useful in the real world because the "atmosphere" of their project-based classrooms was "the same"; the constraints and affordances they had become attuned to were similar, which meant that they could use their school-learned mathematics. This was not to do with the clarity of their learning of mathematical concepts, but with the way they interacted with the broad activity systems of the classroom and the real world. Greeno & MMAP (1998) have described learning and transfer in the following way:

'Learning in this situative view is hypothesized to be becoming attuned to constraints and affordances of activity and becoming more centrally involved in the practices of a community (Lave & Wenger, 1991), and transfer is hypothesized to depend on attunement to constraints and affordances that are invariant or modifiable across transformations of a situation where learning occurred to another situation in which that learning can have an effect (Greeno, Smith & Moore, 1993).' (Greeno & MMAP, 1998, p11).

The Amber Hill students abandoned their use of school mathematics because the number of constraints and affordances to which they became attuned, that were 'invariant across situations' was minimal. The Phoenix Park students were able to use their mathematics because the constraints and affordances to which they became attuned were similar to those of the real world.

Discussion and Conclusion.

I have attempted in this paper to analyze some of the data from my three-year research study, using the perspective of situated cognition, partly to gain a better understanding of the students' behaviour in the two schools and partly to illustrate the utility of such a perspective in doing so. I would now like to describe three key insights that I believe are provided by such a perspective, concerning the nature of knowledge and, leading on from this, the effectiveness of different learning environments and notions of ability.

Lave has posed the question - 'If learning-in-practice is ubiquitous, what are we to make of educational institutions [...] and of failure to learn?' (1993a, p10). I believe this to be a fascinating question that draws a sharp distinction between situated perspectives and other theories of learning. The inability of the Amber Hill students to use school learned methods in GCSE, and in applied and real world situations, could suggest that they spent time in their mathematics classrooms failing to learn. A behaviorist might conclude that they received insufficient opportunities to practice methods, whereas a cognitivist might conclude that they received insufficient opportunity to understand methods in depth. Both of these observations may be accurate, but a situated perspective adds an important, but rarely acknowledged observation — that the Amber Hill students learned a lot during their time in their mathematics classrooms. They learned to become extremely effective in the practices of their school community, interpreting subtle textbook cues and generally engaging in what Birdwhistell, (quoted in McDermott, 1993, p276), calls the 'patterned participation's, systematic dances' of practice. Such a perspective does not preclude the importance of practice or depth of understanding; it includes both of these within a focus on

the broader patterns of participation in communities and the various constraints and affordances — including breadth and depth of understanding — to which students become attuned. Greeno & MMAP (1998) provide an excellent analysis of the way in which situated theory may include, rather than replace, insights gained from cognitive and behaviorist perspectives. It was unfortunate for the students at Amber Hill that becoming effective in their school mathematics classroom did not require the development of mathematical understanding, but this was not a phenomenon for which they could be held responsible, nor one that could be adequately represented as failure to learn.

There is no one situated way of working — both the Amber Hill and Phoenix Park students demonstrated the situated nature of their learning — but recognition of the importance of the learning community and the attunement of students to the practices within those communities carries with it important implications for educational practice. Such perspectives suggest that it is insufficient for students to learn methods clearly, even when they have opportunities to construct (Cobb, Yackel & Wood, 1992) their own mathematical understanding, if they do not also receive opportunities to use mathematics, to choose methods, to change and adapt methods, to discuss and negotiate directions with other students, to interact with systems in the environment (Greeno, 1991) and generally to become attuned to constraints and affordances that are represented in other situations. Such opportunities are not provided only by teaching approaches that have been based on ideas of situativity, or cognitive apprenticeship, many 'reform' classrooms (Stein, Silver & Smith, in press; Fennema et al, 1996; Campbell, 1996; Stocks & Schofield, 1994) provide similar constraints and affordances because these are known to enhance understanding. But a situated perspective suggests that such experiences do not only enhance individual understanding, they provide students with opportunities to engage in practices that are represented and required in everyday life. George's comment, given earlier, presents a stark illustration of the importance of opportunities for discussion and negotiation in the classroom:

> G: I use my own methods. [...] 'Cause when we're out of school yeah, we think, when we're out of school it's social, you're not like in school, it tends to be social, so it would be like too much change to refer back to here. (George, Amber Hill, year 10)

But this comment suggests that the lack of discussion and negotiation that George experienced was important not only because he received less opportunity to derive meaning through a discussion of mathematical concepts, but also because it contributed towards his perceptions of difference. George and his fellow students regarded the classroom and the rest of the world as different communities of practice and this meant that the mathematics they learned at school was of no use to them outside school.

A situated perspective, with its alternative conceptions of what it means to have and to use knowledge, carries with it changed notions of what it means to have ability. If being a knowledgeable person is considered only in terms of individual cognitive structures then notions of ability are straightforward: people merely have more or less of it. The majority of practices in formal educational institutions, particularly those governing assessment, are based implicitly upon such simplistic notions. McDermott (1993), however, has given a careful account of the way in which children are positioned as 'learning disabled' and as part of this account he shows that children have different abilities in different settings, in their interactions with different people and at different

times. Säljö and Wyndhamn (1993) describe the different forms of mathematical knowledge that students produce in different settings and the Amber Hill and Phoenix Park students also showed that their knowledge was co-constituted by the environments in which they worked. Resnick (1993) has suggested that many sociological theories lead to the belief that the main thing people learn in school is how to behave in school, from this perspective the Amber Hill students learned a great deal. Such observations challenge ideas that children's mathematical ability can be located on a one-dimensional scale, that moves from 'none' at one end to 'a lot' at the other and in Lave's situated analysis 'success and failure at learning are viewed not as attributes of individuals, but as specialised social and institutional arrangements' (1993a, p10). This is a non-trivial observation that calls into question the monolithic, high-stakes assessments that are used to rank children and the interpretations that are made of such assessments. Such observations should also shift the locus of blame and responsibility away from the students who attend mathematics classrooms, where it so often rests, to the environments we provide for students and the practices we encourage. In this way situated perspectives can move mathematics education away from the discriminatory practices that produce more failures than successes (Greeno, 1998), towards something considerably more equitable and supportive of social justice.

Lave and Wenger (1991) have described becoming more effective in a community as a process of moving from peripheral to full participation and a change in identity from 'newcomer' to 'old-timer' (Lave and Wenger, 1991, p115). These descriptions pertain to apprenticeship models of learning and highlight some of the important differences between school and apprenticeship communities. Lerman (*in press*) has described some of the motivational differences between being a school student and an apprentice and the differences in relationships formed between apprentice and 'master', as opposed to school student and teacher. Another important difference between the communities of school classrooms and workplaces relates to the established nature of the workplace. Part of what an apprentice learns in the workplace is to talk, act and engage as old-timers do, leading to the focus, which Lave and Wenger propose, upon degrees of continuity and displacement, and the establishment of 'legitimacy'. When the Amber Hill students started their class-taught, textbook approach they were not entering into an established community, but engaging in an evolving community that they shaped themselves. There was no master student that they emulated, for the teacher played a very different role, but still their development of a work identity was central to their behaviour and they engaged in shared participation of an implicit set of school mathematics rules, codes and practices. They did not learn the 'attitudes, knowledge and skills that are valued and useful' (Stein & Brown, 1997, p164) within an established community, but they did learn a set of attitudes, knowledge and skills that were shared with other students and that were specific to the mathematics classroom. Becoming a member of a classroom community is not as well represented by Lave and Wenger's notions of 'legitimate peripheral participation', as becoming a member of a workplace, even a member of the teaching workplace (Stein, Silver & Smith, in press) , because emergent and developing environments are not concerned with legitimacy or peripherally. The students' changing experiences as they moved from year 9 to year 11 may be better represented as moving from inexperienced to experienced participants of their school mathematics community or more generally, by a 'transformation of participation' (Stein & Brown, 1997, p160).

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At Amber Hill school there were important institutional barriers that distinguished the students' experiences in school from the experiences of the rest of their lives. General school rules and practices such as school uniform, timetables, discipline and order contributed to these as well as the esoteric mathematical practices of formalization and rule following. At Phoenix Park the barriers between school and the real world were less distinct: there were no bells at the school, students did not wear uniform, the teachers did not give them orders, they could make choices about the nature and organization of their work and whether they worked or not, mathematics was not presented as a formalised, algorithmic subject and the mathematics classroom was a social arena. The communities of practice making up school and the real world were not inherently different and the constraints and affordances provided by the two situations were similar. Within school the Phoenix Park students did not view mathematics as a formalised and abstract entity that was only useful for school mathematics problems and they did not believe that boundaries (Siskin, 1994) separated their school mathematics knowledge from the rest of their lives. In contrast the Amber Hill students developed a narrow view of mathematics that they regarded as useful only within classroom textbook situations. The students regarded the school mathematics classroom as one 'community of practice' (Lave, 1993b, 1996) and other places, even the school examination hall as different communities of practice. The Amber Hill students' experience of working within such esoteric mathematical environments led them to believe that school and everyday mathematics were of different worlds. Lave has moved away from the idea of boundaries that exist between formal and informal knowledge in order to recognize the overlapping and mutually constituting nature of cognition, but the notion of boundaries or barriers may retain some use, not as a means of separating formal and informal knowledge, but as recognition that the unnatural systems and structures we impose on students in schools — the constraints and affordances to which they become attuned — create perceptions of boundaries and barriers amongst students, even if human cognition is naturally more flexible & dynamic.

The depth of understanding that students develop of mathematical concepts and procedures is extremely important, but the Amber Hill students demonstrated that a focus only upon individual cognitive structures was inadequate. The teachers at Amber Hill explained mathematical methods clearly and the students received opportunities to practice these methods. The students were confident in their use of school methods in the classroom and it is inconceivable to think that they understood none of these methods, yet the students suggested that they were unable to use any of their school mathematical methods in real world situations. Further, the reasons they gave for this were not related to inadequate understanding, but the differences in the constraints and affordances provided by the two environments. This suggests that if we as educators are to understand more about students' use of mathematical knowledge, we need to extend our focus beyond the concepts and procedures that students learn to the practices in which they engage as they are learning them, and the mediation of cognitive forms by the environments in which they are produced.

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